

esupio facile
max 1,5 ore
anche n. 1 facile

FACOLTÀ DI SCIENZE MM.FF.NN.
Corso di laurea in Scienze Biologiche

Prova scritta di Matematica
del 28-04-2010

4-2010

Cognome..... Nome..... N° Matricola.....

1. Enunciare, dimostrare e commentare il teorema di Rolle.

2. Studiare la seguente funzione:

$$f(x) = 4 - \frac{x^2 - 6}{x^2 - 9}$$

OK 3. Calcolare la derivata delle seguenti funzioni:

$$f(x) = 2x + (e^{2x} \cos(2x)) + 6e^{2x} \sin(2x);$$

$$g(x) = 1 + 2x + 3x^{4 \cos x}$$

4. Calcolare il seguente integrale indefinito:

$$\int [4(\lg x)^2 x^3 + 3] dx. \quad \text{DA FORA}$$

OK 5. Calcolare il seguente limite:

$$\lim_{x \rightarrow +\infty} \left[\frac{\sqrt{3x^4 - 5} - \sqrt{3x^4 + 5}}{3} + 3 \right].$$

$$y = 4 - \frac{x^2 - 6}{x^2 - 9}$$

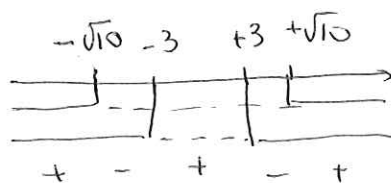
(D) $x^2 - 9 \neq 0 \Rightarrow x \neq \pm 3$

$$]-\infty; -3[\cup]-3; 3[\cup]3; +\infty[$$

(S) $\frac{4(x^2 - 9) - (x^2 - 6)}{x^2 - 9} \geq 0 \Rightarrow \frac{4x^2 - 36 - x^2 + 6}{x^2 - 9} \geq 0 \Rightarrow \frac{3x^2 - 30}{x^2 - 9} \geq 0$

$$3x^2 - 30 \geq 0 \Rightarrow 3x^2 - 30 = 0 \Rightarrow x^2 = \frac{30}{3} \Rightarrow x^2 = 10 \Rightarrow x_{1/2} = \pm \sqrt{10} \approx \pm 3,16$$

$$x^2 - 9 \geq 0 \Rightarrow x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x_{1/2} = \pm 3$$



(Int.)

osservo che il cambio di segno a $\pm \sqrt{10}$ che $\in D$ identifica i p.t. di int. del piano x $A(-\sqrt{10}; 0)$ $B(+\sqrt{10}; 0)$

Int. A $SSP = Y \Rightarrow y = 4 - \frac{0^2 - 6}{0^2 - 9} = 4 - \frac{6}{-9} = \frac{36 - 6}{9} = \frac{30}{9} \approx 3,33$ $C(\sqrt{10}, \frac{30}{9})$

(L)

$$\lim_{x \rightarrow \pm \infty} 4 - \frac{x^2 - 6}{x^2 - 9} = \lim_{x \rightarrow \pm \infty} 4 - \frac{x^2(1 - \frac{6}{x^2})}{x^2(1 - \frac{9}{x^2})} = \lim_{x \rightarrow \pm \infty} 4 - \frac{1 - \frac{6}{x^2}}{1 - \frac{9}{x^2}} = 4 - 1 = 3 \quad A.O.$$

$$\lim_{x \rightarrow -3^-} 4 - \frac{x^2 - 6}{x^2 - 9} = \lim_{x \rightarrow -3^-} 4 - \frac{(-3)^2 - 6}{0^+} = 4 - \frac{9 - 6}{0} = 4 - \frac{3}{0} = 4 - \infty = -\infty \quad A.V.$$

$$\lim_{x \rightarrow -3^+} 4 - \frac{x^2 - 6}{x^2 - 9} = \lim_{x \rightarrow -3^+} 4 - \frac{9 - 6}{0^-} = 4 - \frac{3}{0^-} = 4 - (-\infty) = 4 + \infty = +\infty \quad A.V.$$

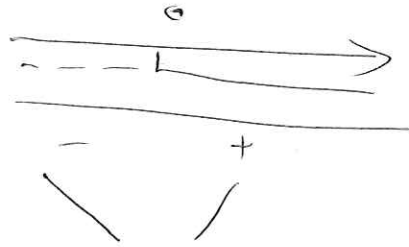
$$\lim_{x \rightarrow +3^-} f(x) = +\infty \quad A.V. \quad \parallel \quad \lim_{x \rightarrow +3^+} f(x) = -\infty \quad A.V.$$

Non alcuni pezzi e non quelli sistemati!

$$y' = \frac{-(2x)(x^2-9) - (-x^2-6)2x}{(x^2-9)^2} = \frac{-2x^3 + 18x + 2x^3 - 12x}{(x^2-9)^2} = \frac{6x}{(x^2-9)^2} \geq 0$$

$$\Leftrightarrow 6x \geq 0 \Rightarrow x \geq 0$$

$$(x^2-9)^2 \geq 0 \quad \forall x \in \mathbb{D}$$



$$x=0 \in \mathbb{D} \Rightarrow x=0 \Rightarrow y = \frac{30}{9}$$

$C(0; \frac{30}{9})$ è l'unico

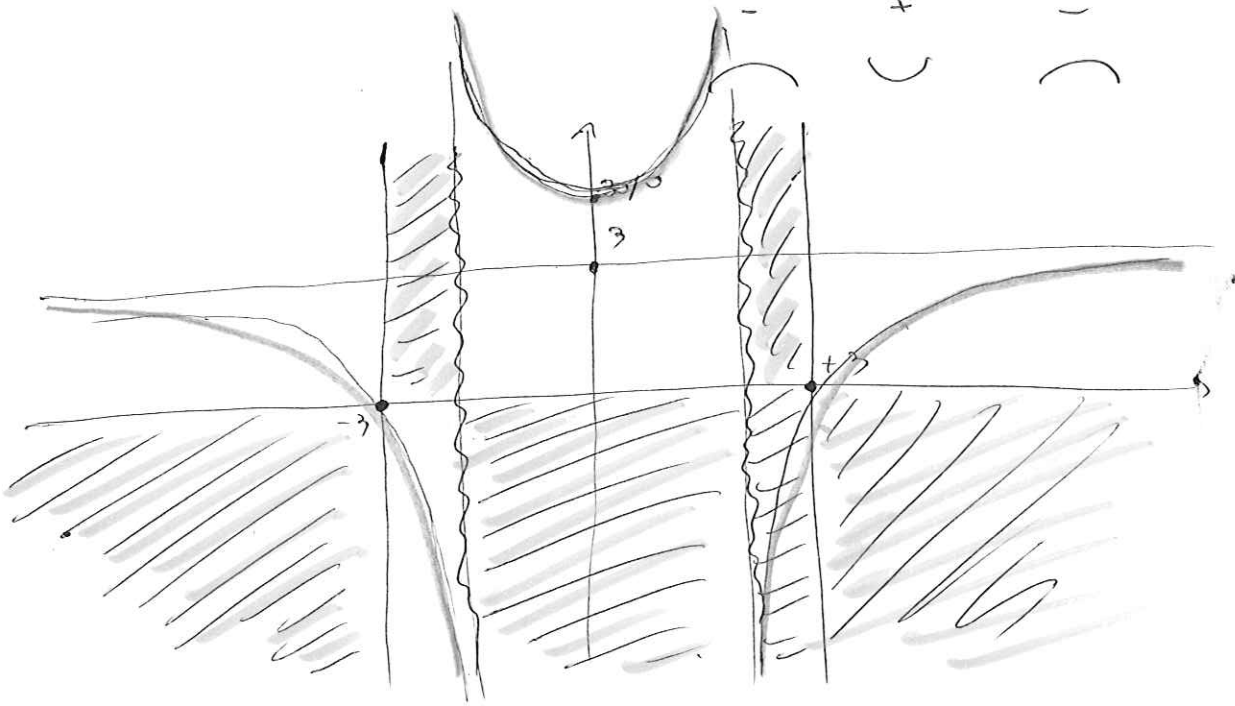
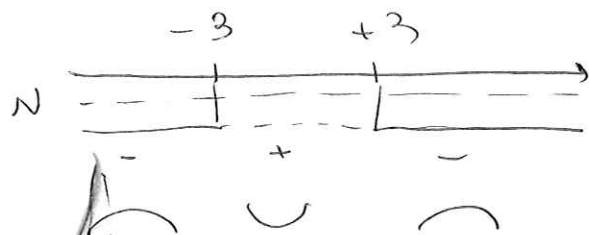
$$y'' = \frac{6(x^2-9)^2 - 6x \cdot 2(x^2-9) \cdot 2x}{(x^2-9)^4} = \frac{(x^2-9)[6(x^2-9) - 24x^2]}{(x^2-9)^4} =$$

$$= \frac{6x^2 - 54 - 24x^2}{(x^2-9)^3} = \frac{-18x^2 - 54}{(x^2-9)^3} \geq 0$$

$$\text{NUM. } -18x^2 - 54 = 0 \quad \Delta = 0 - 4 \cdot (-18) \cdot (-54) < 0 \Rightarrow \exists x_{1,2} \in \mathbb{R}$$

è sempre negativo

$$\text{DENOM. } (x^2-9)^3 > 0 \Rightarrow x^2-9 > 0 \quad x_{1,2} = \pm 3$$



$$f(x) = 2x + e^{2x} \cos(2x) + 6e^{2x} \sin(2x)$$

$$\begin{aligned} f'(x) &= 2 + \underline{2e^{2x} \cos(2x)} + \underline{2e^{2x} \sin(2x)} + 12e^{2x} \sin(2x) + 12e^{2x} \cos(2x) = \\ &= 2 + 14e^{2x} \cos(2x) + 10e^{2x} \sin(2x) \end{aligned}$$

$$g(x) = 1 + 2x + 3x^{4\cos x}$$

$$g'(x) = 2 + 3 \underbrace{(x^{4\cos x})}_{(A)}$$

$$(A) \quad y = x^{4\cos x}$$

$$\log y = \log(x^{4\cos x})$$

$$\log y = 4\cos x \log x$$

$$(\log y)' = (4\cos x \log x)'$$

$$\frac{y'}{y} = -4\sin x \log x + \frac{4\cos x}{x}$$

$$y' = \left(-4\sin x \log x + \frac{4\cos x}{x} \right) \cdot x^{4\cos x}$$

$$\Rightarrow g'(x) = 2 + 3x^{4\cos x} \left(-4\sin x \log x + \frac{4\cos x}{x} \right)$$

$$\int 4(\log x)^2 x^3 + 3 \, dx = 4 \int (\log x)^2 x^3 \, dx + 3 \int dx = 4 \underbrace{\int \log^2 x \cdot x^3 \, dx}_{(A)} + 3x + C$$

$$(A) \int \log^2 x \cdot x^3 \, dx = \left\{ \begin{array}{l} f'(x) = x^3 \Rightarrow f(x) = \frac{x^4}{4} \\ g(x) = \log^2 x \Rightarrow g'(x) = \frac{2 \log x}{x} \end{array} \right.$$

$$= \int \frac{x^4}{4} \cdot \log^2 x - \int \frac{x^4}{4} \cdot \frac{2 \log x}{x} \, dx =$$

$$= \frac{x^4}{4} \log^2 x - \frac{1}{2} \int x^3 \log x \, dx =$$

per parti: $\left\{ \begin{array}{l} g(x) = \log x \Rightarrow g'(x) = \frac{1}{x} \\ f(x) = x^3 \Rightarrow f'(x) = \frac{x^4}{4} \end{array} \right.$

$$= \frac{x^4}{4} \log^2 x - \frac{1}{2} \left(\frac{x^4}{4} \log x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx \right) =$$

$$= \frac{x^4}{4} \log^2 x - \frac{1}{8} x^4 \log x + \frac{1}{8} \int x^3 \, dx =$$

$$= \left[\frac{x^4}{4} \log^2 x - \frac{1}{8} x^4 \log x + \frac{x^4}{32} + C \right] = A$$

VERIFICA: derivo il risultato $A' = \frac{1}{4} \left(4x^3 \log^2 x + \frac{x^4 2 \log x}{x} \right) - \frac{1}{8} \left(4x^3 \log x + \frac{x^4}{x} \right) + \frac{4x^3}{32}$

$$= x^3 \log^2 x + \frac{1}{2} x^3 \log x - \frac{1}{2} x^3 \log x - \frac{1}{8} x^3 + \frac{1}{8} x^3 = x^3 \log^2 x \quad \checkmark$$

VERIFICATO (A)

$$\Rightarrow \int 4(\log x)^2 x^3 + 3 \, dx = 4 \left(\frac{x^4}{4} \log^2 x - \frac{1}{8} x^4 \log x + \frac{x^4}{32} \right) + 3x =$$

$$= \left[x^4 \log^2 x - \frac{1}{2} x^4 \log x + \frac{x^4}{8} + 3x + C \right]$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{3x^4-5} - \sqrt{3x^4+5}}{3} + 3 \quad \text{resultado} =$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{3x^4-5} - \sqrt{3x^4+5}) (\sqrt{3x^4-5} + \sqrt{3x^4+5})}{\sqrt{3x^4-5} + \sqrt{3x^4+5}} \cdot \frac{1}{3} + 3 =$$

$$= \lim_{x \rightarrow +\infty} \frac{(3x^4-5) - (3x^4+5)}{\sqrt{3x^4-5} + \sqrt{3x^4+5}} \cdot \frac{1}{3} + 3 = \lim_{x \rightarrow +\infty} \frac{3x^4-5-3x^4-5}{\sqrt{3x^4-5} + \sqrt{3x^4+5}} \cdot \frac{1}{3} + 3 =$$

$$= \lim_{x \rightarrow +\infty} \frac{-10}{\infty} \cdot \frac{1}{3} + 3 = 0 + 3 = 3$$